The MILP model for the sub-problem of re-route

First, all the notations of the parameters and variables used in the model are listed in Table 1.

Table 1: Notations of the online food delivery problem

Variable	Explanation
C_v	Set of active customers which are assigned to courier v.
$C_v^{'}$	Set of active customers for which $v \in V$ has already picked up at least one item for this customer.
I_c	Set of restaurants where customer $c \in C$ placed the order.
γ	A penalty coefficient($\gamma \geq 0$).
t_c^{max}	Promised delivery time for customer c.
t_i^{min}	Estimated time for restaurant i to finish the preparation, $i \in R$.
δ^{max}	Maximum allowable penalized time beyond t_c^{max} deliver to each customer.
s_i	Services intervals in restaurants and customers' locations, $i \in \{R \cup C\}$.
$t_v^{'}(t_v^{''})$	Start (end) time of courier $v \in V$'s shift.
$\hat{w}_j$	Quantity of items picked up from location $j \in I_c(\hat{w}_j > 0)$ or dropped off at location $c \in C(\hat{w}_j < 0)$.
$\bar{w_v}$	Number of items already being carried by courier $v \in V$ at $t_v^{'}$.
W_v	Maximum payload capacity of courier $v \in V$.
Δ_v^0	Initial node for courier $v \in V$.
Δ^*	Dummy (virtual) final node for all couriers; $\Delta^* = 0$.
Δ_v^+	Set of all nodes to which courier $v \in V$ may travel; $\Delta_v^+ \subseteq \{C \cup \{\cup_{c \in C} I_c\} \cup \Delta^*\}$.
Δ_{vj}^-	Set of all nodes from which courier $v \in V$ may travel to node $j \in \Delta_v^+$; $\Delta_{vj}^- \subseteq \{\Delta_v^0 \cup \{\cup_{c \in C} I_c \setminus j\} \cup \{C \setminus j\}\}$.
τ_{vij}	Time required by courier $v \in V$ to travel to node $j \in \Delta_v^+$ from node $i \in \Delta_{vj}^-$, calculated by the distance between i and j dividing average speed $\overline{v}$ of v.
x_{vij}	Binary decision variable equals 1 if courier $v \in V$ travels to node $j \in \Delta_v^+$ from node $i \in \Delta_{vj}^-$; 0 otherwise.
y_{vi}	Binary decision variable equals 1 if courier $v \in V$ picks up an item from node $i \in I_c$; 0 otherwise.
t_{vi}	Continuous decision variable representing the time courier $v \in V$ visits node $i \in \Delta_v^+$.
w_{vij}	Continuous decision variable representing the quantity of items in possession of courier $v \in V$ on the route to node $j \in \Delta_v^+$ after visiting node $i \in \Delta_{vj}^-$.

Then, the MILP model is established as follows:

$$\epsilon_v = \max \sum_{c \in C_v} ((t_c^{max} - t_{vc}) - \gamma * (t_{vc} - t_c^{max})^+) \tag{1}$$

$$\text{s.t.} \sum_{i \in \Delta_{vc}^-} x_{vic} = 1, \forall c \in \{C_v \cup C_v^{'}\}, \tag{2}$$

$$\sum_{i \in I_c} y_{vi} = |I_c| \sum_{i \in \Delta_{vc}^-} x_{vic}, \forall c \in C_v, \tag{3}$$

$$t_{vc} \leq t_c^{max} + \delta^{max}, \forall c \in C_v \tag{4}$$

$$t_{vj} \leq (\max_{c \in C_v}\{t_c^{max}\} + \delta^{max}) \sum_{i \in \Delta_{vj}^-} x_{vij}, \forall j \in \Delta_v^+, \tag{5}$$

$$t_{vi} \leq t_{vc} - (s_i + \tau_{vic}) y_{vi}, \forall c \in C_v, i \in I_c, \tag{6}$$

$$t_{vi} \geq t_i^{min} y_{vi}, \forall c \in C_v, i \in I_c, \tag{7}$$

$$\begin{aligned} t_{vj} \geq t_{vi} + s_i + \tau_{vij} - (\max_{c \in C_v}\{t_c^{max}\} + \delta^{max} + s_i + \tau_{vij})(1 - x_{vij}), \\ \forall j \in \Delta_v^+, i \in \{\Delta_{vj}^- \setminus \Delta_v^0\}, \end{aligned} \tag{8}$$

$$w_{vij} \leq W_v, \forall j \in \Delta_v^+, i \in \Delta_{vj}^-, \tag{9}$$

$$w_{v,\Delta_v^0,j} = (\bar{w_v} + \hat{w_j}) x_{v,\Delta_v^0,j}, \forall j \in \Delta_v^+, \tag{10}$$

$$w_{vjk} \geq \sum_{i \in \Delta_{vj}^- : i \neq k} w_{vij} + \hat{w}_k x_{vjk} - W_v(1 - x_{vjk}), \forall k \in \Delta_v^+, j \in \{\Delta_{vk}^- \setminus \Delta_v^0\}, \tag{11}$$

$$y_{vk} = \sum_{i \in \Delta_{vk}^-} x_{vik}, \forall c \in C_v, k \in I_c, \tag{12}$$

$$y_{vi} = 1, \forall i \in I_c, c \in C_v^{'}, \tag{13}$$

$$\sum_{j \in \Delta_v^+} x_{v,\Delta_v^0,j} = 1 \tag{14}$$

$$\sum_{i \in \Delta_{v,\Delta^*}^-} x_{v,i,\Delta^*} = 1 \tag{15}$$

$$\sum_{i \in \Delta_{vj}^-} x_{vij} = \sum_{k \in \{\Delta_v^+ : j \in \Delta_{vk}^-\}} x_{vjk}, \forall j \in \{\Delta_v^+ \setminus \Delta^*\}, \tag{16}$$

$$t_{vj} \geq (t_v^{'} + \tau_{v,\Delta_v^0,j}) x_{v,\Delta_v^0,j}, \forall j \in \Delta_v^+, \tag{17}$$

$$t_{v,\Delta^*} \leq t_v^{''} \tag{18}$$

$$t_{vi} \geq 0, \forall v \in V, i \in \Delta_v^+, \tag{19}$$

$$w_{vij} \geq 0, \forall j \in \Delta_v^+, i \in \Delta_{vj}^-, \tag{20}$$

$$x_{vij} \in \{0,1\}, \forall j \in \Delta_v^+, i \in \Delta_{vj}^-, \tag{21}$$

$$y_{vi} \in \{0,1\}, \forall c \in C_v, i \in I_c. \tag{22}$$

In the objective function (1), we consider the maximum early time and the minimum delay time to obtain higher customer satisfaction. Because the impact of early delivery on customers is weaker than late delivery, we add a penalty coefficient to the delay time.

Constraints (2) ensure the customer will be served once only. Constraints (3) state that the courier must complete the corresponding delivery to the customer.

Constraints (4)–(8) state the timing relationships. Specifically, constraints (4) and (5) ensure that the delivery to customer must occur before the maximum allowable time $t_c^{max} + \delta^{max}$. Constraints (6) guarantee that the t_{vj} equals to 0 if the courier v does not visit node j, otherwise, the constraints will be non-binding. Constraints (7) represent that if courier v delivery for customer c, then the visit time must exceed the earliest time that the order is ready for picking up. Constraints (8) state that if v travels from node i to node j, the arrival time at j is greater than the arrival time at i plus the service time s_i plus the travel time τ_{vij}, otherwise, the constraints will be non-binding.

Constraints (9)-(11) characterize the payload capacity requirements. The quantity of items in possession of each courier is limited by constraints (9) to maximum payload capacity. The courier's payload on the route from initial node Δ_v^0 to node j are defined by constraints (10). Similarly, constraints (11) define the payload of courier as they travel from node j to node k.

The relationship between decision variables y and x are given by constraints (12). Constraints (13) capture existing assignments. Specifically, if any item for a particular customer has already been picked up, then this customer must now be tied to this courier. Constraints (14) and (15) require the courier must leave from their initial location and terminate their route at the dummy ending node. Meanwhile, if the courier visited one node, he must leave from the same node, as guaranteed by constraints (16). Constraints (17) and (18) define the earliest and latest time of all delivery, respectively. Finally, constraints (19)-(22) give the boundary restrictions for decision variables.