

**An improved approach of winter wheat yield estimation by jointly  
assimilating remotely sensed leaf area index and soil moisture into the  
WOFOST model**

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## 1. Ensemble Kalman Filter

For the implementation of the EnKF, we based on our previous research of [Huang et al. \(2016\)](#) and [Zhuo et al. \(2022\)](#). The core part of the EnKF is the calculation of the Kalman gain matrix,

$$y = Hx_t + \varepsilon \quad (1)$$

$$x_t = Ax_{t-1} + v \quad (2)$$

Where  $y$  is the observation vector,  $\varepsilon$  and  $v$  are Gaussian random error vector with a mean of zero, and  $H$  is the observation operator related to  $y$ , it can be taken as an identity matrix in this study.  $A$  represents a state-transition model that links  $x_t$  and  $x_{t-1}$ , in the CDMA system it represents crop model. And the forecast of  $x_t$  at  $t=k$  is Gaussian with mean  $x_{t=k}^f$  and error covariance  $P_{t=k}^f$  can be calculated as Eqs. (3) – (4):

$$x_{t=k}^a = x_{t=k}^f + K(y - Hx_{t=k}^f) \quad (3)$$

$$P_{t=k}^a = (I - KH)P_{t=k}^f \quad (4)$$

Where  $f$  and  $a$  are indices of the prior and posterior estimates, respectively,  $I$  is the identity matrix, and  $K$  is the Kalman gain matrix defined as Eq. (5)

$$K = P_{t=k}^f H^T (HP_{t=k}^f H^T + R)^{-1} \quad (5)$$

When solving the Kalman gain, [Houtekamer and Mitchell \(2001\)](#) suggest calculating  $P_{t=k}^f H^T$  and  $HP_{t=k}^f H^T$  directly from the ensemble members, rather than calculating each element of Eq. (5):

$$P^f H^T = (N_e - 1)^{-1} \sum_{n=1}^{N_e} (x_n^f - \bar{x}^f)(Hx_n^f - H\bar{x}^f)^T \quad (6)$$

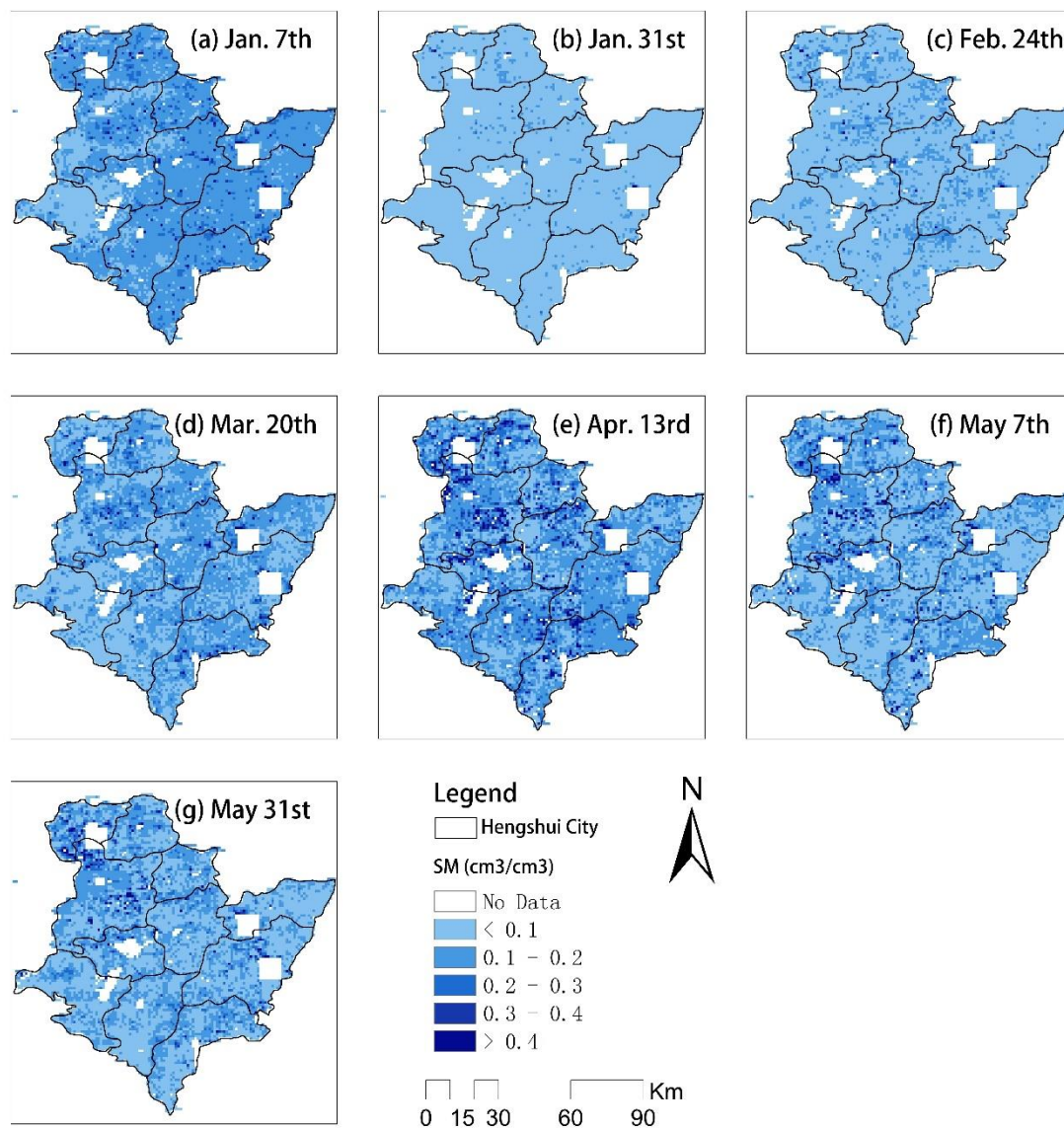
$$HP^f H^T = (N_e - 1)^{-1} \sum_{n=1}^{N_e} (Hx_n^f - H\bar{x}^f)(Hx_n^f - H\bar{x}^f)^T \quad (7)$$

Where  $N_e$  is the number of ensemble members,  $n$  is a running index for ensemble member, and  $\bar{x}^f$  represents the ensemble mean calculated as Eq. (8).

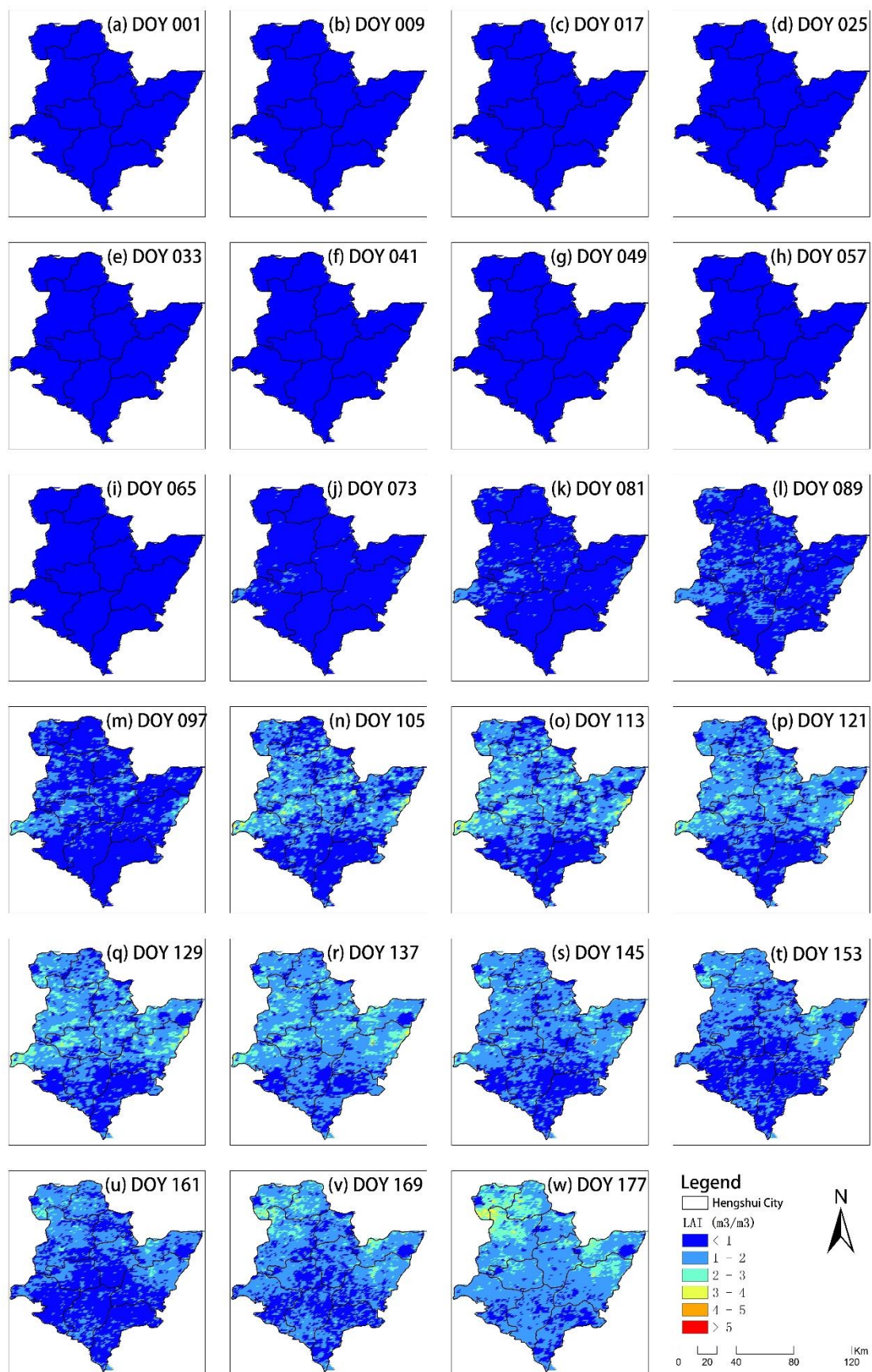
$$\bar{x}^f = N_e^{-1} \sum_{n=1}^{N_e} x_n^f \quad (8)$$

$$H\bar{x}^f = N_e^{-1} \sum_{n=1}^{N_e} Hx_n^f \quad (9)$$

## 2. Supplementary Figures



**Figure S1.** SM<sub>SMAP</sub> images of Hengshui city in 2017 during winter wheat growing season

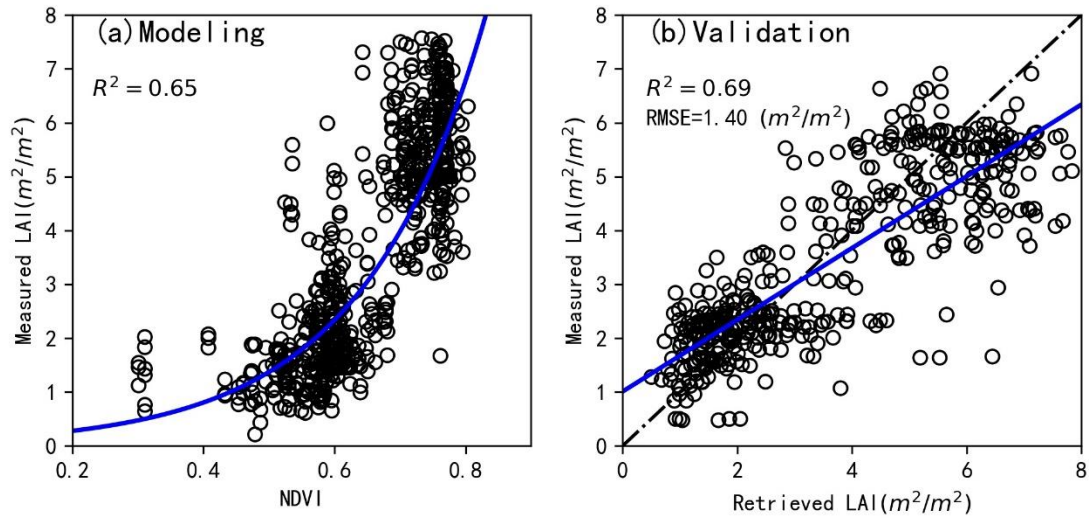


**Figure S2.** LAI<sub>MODIS</sub> images of Hengshui city in 2017 during winter wheat growing season

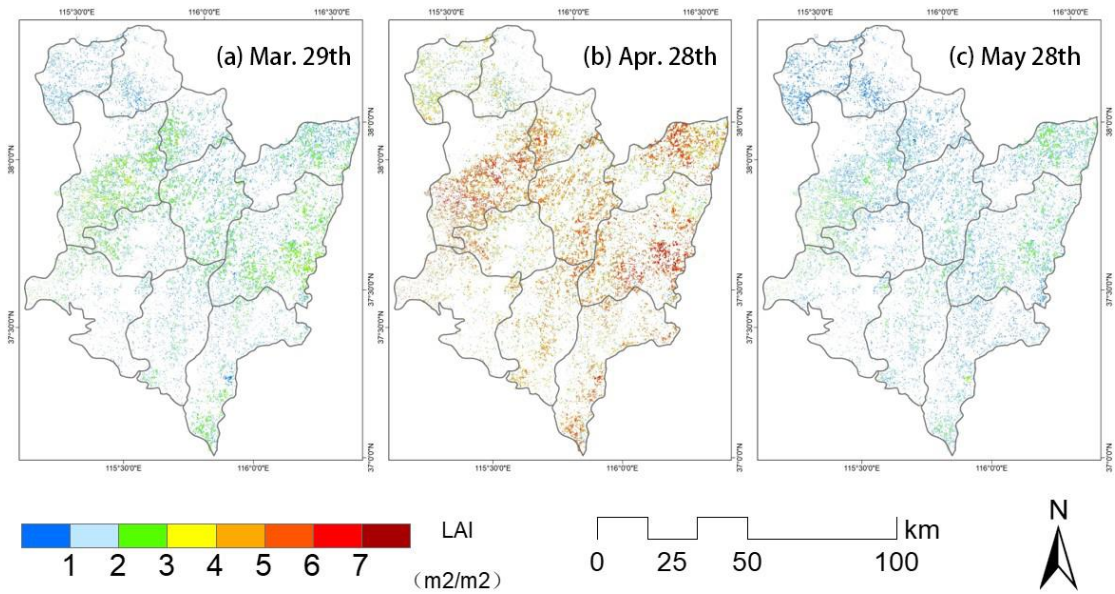


The fitting results of LAI and NDVI are shown as formula (1):

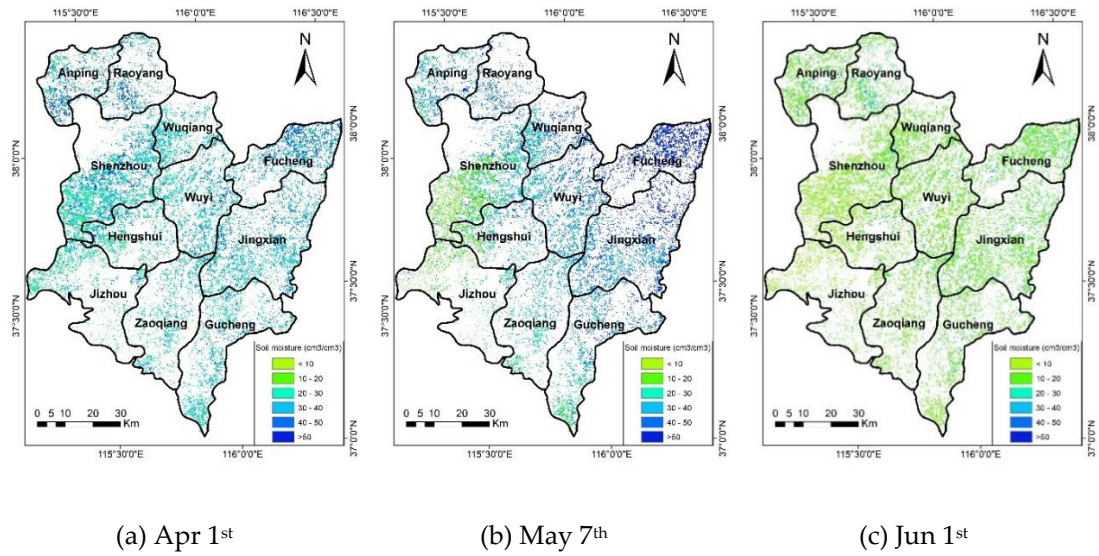
$$LAI = -1/0.304 * \ln [(0.901 - NDVI)/0.717] \quad (1)$$



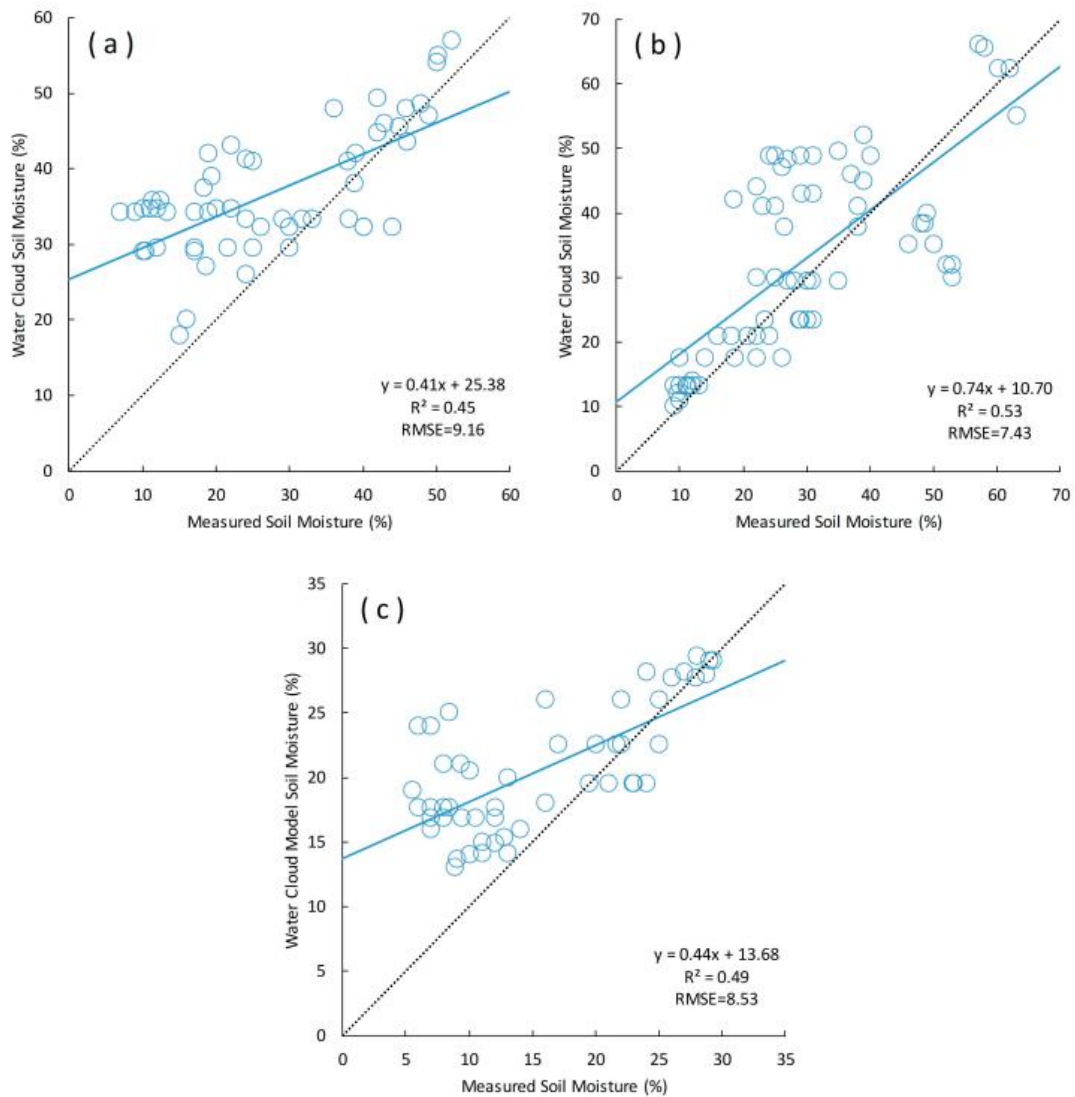
**Figure S3.** (a) Modeling of measured LAI and NDVI (b) Validation of retrieved LAI results



**Figure S4.** LAI<sub>s2</sub> images of Hengshui city in 2017 during winter wheat growing season



**Figure S5.** Maps of SM in the study area retrieved from the water cloud model (a) 1 April, (b) 7 May and (c) 1 Jun. (Zhuo et al., 2019)



**Figure S6.** Validation of the retrieved SM from the water cloud model (Zhuo et al., 2019)

## References

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